Revising D3- Curve Matching

D3: I can use information about f, f', and/or f" to identify and/or draw accurate graphs of the other functions.

Basic Preparation

1. Did you do the written practice?
2. Did you do the WeBWorK?
3. Go back to your notes, any handouts from class, the desmos activities, WeBWorK and any written practice you were able to use to prepare. Compare this to your quiz/homework.
4. Do you understand what your mistake was? If so, briefly describe what the mistake is below. If you are unsure, please go to the Calculus Center and work with a tutor until you can describe what your mistake was.

Metacognition

Now, *WHY* did you make the mistake? Answering this question is asking you to think about HOW you think about math (metacognition). Spending time here will help you become more efficient at learning math and is therefore worth the time!

1. Was your incorrect answer due to
   (a) not understanding a concept;
   (b) an error in logical reasoning (e.g., used the correct theorem/test but made the wrong conclusion, used a theorem/test/technique when it did not apply);
   (c) being careless (e.g. not reading directions, not answering the question completely, making arithmetic or basic algebra errors);
   (d) not knowing how to start or formulate an approach to the problem;
   (e) others?

   Briefly describe why your answer was incorrect:

2. What helped you recognize your mistake(s). Here are some examples: the course notes, the textbook, homework or conversations from the Calculus Center. In other words, which strategies for identifying mistakes work well for you and will help you in the future?

3. Rework the ENTIRE PROBLEM. Rewrite your solution from start to finish, carefully fixing the mistake(s) you diagnosed above. By doing the entire problem over again, you can make sure you fix your mistake and better understand the point of the exercise.

4. Describe (in detail) what you have done in order to learn from your mistake(s) and prepare for your next attempt. Did you read the textbook or class notes? Did you look at examples and/or work problems on your own or with your tutor/classmate/instructor, and if so, which problems? Did you take a different approach than listed here? (Again, the point of this isn't just to look at what you did on this problem, but how can you learn from this and be more likely to meet expectations on future assignments on the first try.)

Where topic was first introduced: Modules 4 and 5

Videos:

- Video on information about the derivative from the original function
  `https://www.youtube.com/watch?v=SYyecwhLBm8&feature=youtu.be`
- Video on information about the original function from the derivative
  `https://youtu.be/IpxR9qFeA0s`

Derivative Relationships:

- If $f > 0$, then the output values of the function are above the x-axis.
- If $f = 0$, then the output values of the function are on the x-axis.
- If $f < 0$, then the output values of the function are below the x-axis.
- If $f' > 0$, then the output values of the function are increasing.
- If $f' = 0$, then the output values of the function are neither increasing nor decreasing.
- If $f' < 0$, then the output values of the function are decreasing.
- If $f'' > 0$, then the output values of the function are concave up.
- If $f'' = 0$, then the output values of the function are neither concave up nor concave down.
- If $f'' < 0$, then the output values of the function are concave down.

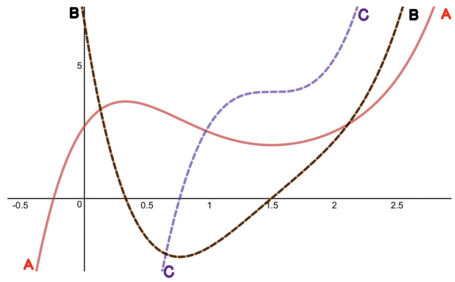
Favorite Mistakes on Curve Matching Problems:

- Make sure your answers convince you and the grader that you fully understand the derivative relationships above- not that you can just say those words, but that you can apply them to specific situations. One way to do this is to give specific x-values. If your answer could be cut and pasted to another question without changing anything, you probably didn't include enough specific detail.
- Don't use the word "IT"!!! Did you mean the function, the derivative or the second derivative? Did you mean output values or slopes?
- While it's possible to use process of elimination, does that response do the best job of convincing you and us that you understand the relationships above?
- If you are ever talking about increasing slopes or decreasing slopes, make sure that's what you mean to say. If the parent function has increasing slopes, then you are also talking about concavity. For example, if $f(x) = x^2$, f is always concave up so the slopes of f are always increasing- think about tangent lines, and for negative x-values the slopes of tangent lines are negative and steep, but as x increases, the slopes become less steep until $x = 0$ and the slope of the tangent line is horizontal, and then positive and increasingly steep as x increases further. Some students correlate increasing slopes with positive first derivatives instead of second derivatives. Again, looking at x^2, the first derivative is negative and then positive around $x = 0$ but the **slopes** are always increasing.
- Writing things like f is increasing on $[-\infty, -4]$. This would mean that the graph f is increasing AT infinity and at $x = -4$. We can never be **AT** infinity. And, if f is changing direction when $x = -4$, then f is probably NOT increasing when $x = -4$ so we shouldn't include that either. Use $(-\infty, -4)$ instead.

- Not using concavity to graphically interpret the second derivative. For the question about which graph has a positive second derivative, the concavity interpretation is really useful.
- Assuming that a graph is of x^2 just because the graph is concave up. Focus on the relationships rather than trying to reverse engineer the equations.
- Not giving a conclusive argument. It is not clear that just because two graphs "mirror each other" or "are both concave up" that one would be the first derivative and one the parent, for example. A fail-safe strategy on these problems is to list all the intervals of positive/negative/neither, increase/decrease/neither, and concave up/down/neither for EACH graph. Then you can discuss the relationships between f, f' and f" by matching intervals of positive to increasing or concave up etc.
- Including x values where a function is neither concave up nor down in an interval of concave up-ness... Hint: use tangent lines to confirm if a function is concave up or concave down. If the tangent line is above a function at an x value, then the function is concave down there.

Example:

Below are the graphs of a position function, $s(t)$, a velocity function, $v(t)$, and an acceleration function, $a(t)$, with respect to time, t.



Graph A

- Graph A has 0 values when $x = -0.2$, positive values for $x > -0.2$ and negative values for $x < -0.2$.
- Graph A is neither increasing nor decreasing when $x = 0.3$, $x = 1.5$, is increasing on $(-\infty, 0.3)$ and $(1.5, \infty)$, and is decreasing on $(0.3, 1.5)$.
- Graph A is neither concave up nor concave down when $x = 0.75$, is concave up for $x > 0.75$ and concave down for $x < 0.75$.

Graph B

- Graph B has 0 values when $x = 0.3$ and $x = 1.5$, positive values for $x < 0.3$ and $x > 1.5$ and negative values for $(0.3, 1.5)$.
- Graph B is neither increasing nor decreasing when $x = 0.75$, is increasing on $(0.75, \infty)$, and is decreasing on $(-\infty, 0.75)$.
- Graph B is possibly always concave up (it looks pretty flat when $x = 1.5$ though). Safer to say Graph B is never concave down.

Graph C

- Graph C has 0 values when $x = 0.75$, positive values for $x > 0.75$ and negative values for $x < 0.75$.
- Graph C is neither increasing nor decreasing when $x = 1.5$, is increasing on $(-\infty, 1.5)$ and $(1.5, \infty)$, and is decreasing never.
- Graph C is neither concave up nor concave down when $x = 1.5$, is concave up for $x > 1.5$ and concave down for $x < 1.5$.

Because Graph C has 0 values when $x = 0.75$, positive values for $x > 0.75$ and negative values for $x < 0.75$ and Graph B is neither increasing nor decreasing when $x = 0.75$, is increasing on $(0.75, \infty)$, and is decreasing on $(-\infty, 0.75)$, Graph B's derivative must be Graph C.

Because Graph B has 0 values when $x = 0.3$ and $x = 1.5$, positive values for $x < 0.3$ and $x > 1.5$ and negative values for $(0.3, 1.5)$ and Graph A is neither increasing nor decreasing when $x = 0.3$, $x = 1.5$, is increasing on $(-\infty, 0.3)$ and $(1.5, \infty)$, and is decreasing on $(0.3, 1.5)$, Graph A's derivative must be Graph B.

Because Graph C has 0 values when $x = 0.75$, positive values for $x > 0.75$ and negative values for $x < 0.75$ and Graph A is neither concave up nor concave down when $x = 0.75$, is concave up for $x > 0.75$ and concave down for $x < 0.75$, Graph A's second derivative must be Graph C.

Thus the parent function (position, $s(t)$) is Graph A, the first derivative (velocity, $v(t)$) is Graph B, and the second derivative (acceleration, $a(t)$) is Graph C.

Prepare for revision:

First, reflect on your mistake and the correct solution and what you learned: fill in the blanks "I used to think ________________but now I think ________________because I learned ________________."

Open a Desmos window and define a function $f(x) =$ (your choice) and graph that function. Then ask desmos to graph $f'(x)$ and $f''(x)$ (you can literally type in $f'(x)$ and desmos will graph the derivative.) Then, match intervals of increase/decrease and concave up/down with intervals of positive and negative to argue which graph is the derivative or second derivative of the other.

Some sample functions to try (please modify to make more practice!!!):

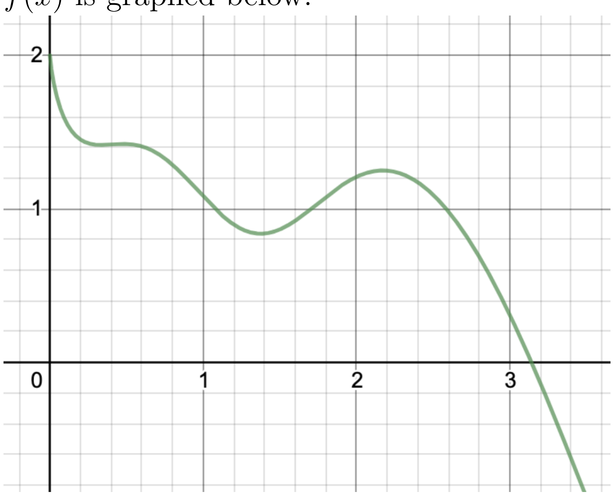
1. $.5x^4 - 2x^3 + x^2 - x + 3$

2. $\sin(x) \ln x + 3$

3. $e^{\sin(x)}$

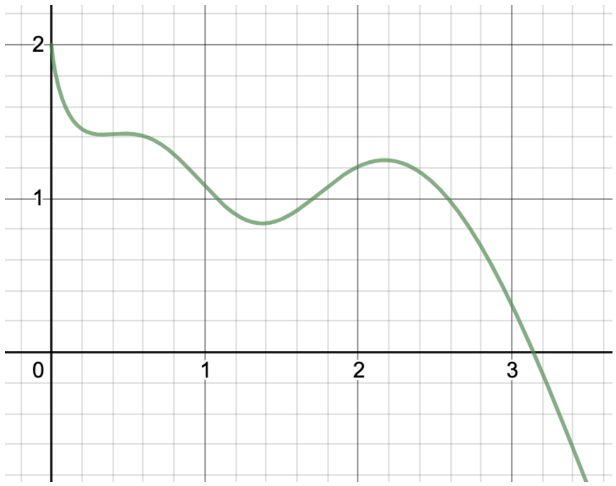
4. $\dfrac{10\,|\cos(x)|\ln(x)}{\sqrt{x^2+4}}$

$f(x)$ is graphed below.



Questions for interpreting the graph as being of the parent function:

- List an x value where the parent function $f(x)$ is A) increasing B) decreasing C) Neither or explain why this is not possible. How do you know?
- List an x value where $f'(x)$ is A) positive B) negative C) Neither or explain why this is not possible. How do you know?
- List an x value where the parent function is A) concave up B) concave down C) Neither or explain why this is not possible. How do you know?
- List an x value where $f''(x)$ is A) positive B) negative C) Neither or explain why this is not possible. How do you know?



Questions for the graph being interpreted as the first derivative:

- List all intervals or x-values where the first derivative is positive, negative, or neither.
- List all intervals where the parent function would be increasing, decreasing, or neither.
- List all intervals where the first derivative is increasing, decreasing, or neither.
- List all intervals where the parent function is concave up, down, or neither.

Questions for the graph being interpreted as the second derivative:

- List all intervals where the second derivative is positive, negative, or neither.
- List all intervals where the parent function is concave up, down, or neither.
- List all intervals where the first derivative is increasing, decreasing, or neither.